Geometry,
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SYMMETRY GROUPS, CONSERVATION LAWS AND GROUP–INVARIANT SOLUTIONS OF THE MEMBRANE SHAPE EQUATION

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Abstract. The six-parameter group of three dimensional Euclidean motions is recognized as the largest group of point transformations admitted by the membrane shape equation in Mongé representation. This equation describes the equilibrium shapes of biomembranes being the Euler-Lagrange equation associated with the Helfrich curvature energy functional under the constraints of fixed enclosed volume and membrane area. The conserved currents of six linearly independent conservation laws that correspond to the variational symmetries of the membrane shape equation and hold on its smooth solutions are obtained. All types of non-equivalent group-invariant solutions of the membrane shape equation are identified via an optimal system of one-dimensional subalgebras of the symmetry algebra. The reduced equations determining these group-invariant solutions are derived. Special attention is paid to the translationally-invariant solutions of the membrane shape equation.

1. Introduction

In aqueous solution, amphiphilic molecules (e.g., phospholipids) may form bilayers, the hydrophilic heads of these molecules being located in both outer sides of the bilayer, which are in contact with the liquid, while their hydrophobic tails remain at the interior. In many cases the bilayer form a closed membrane – that is a vesicle. Vesicles constitute a well-defined and sufficiently simple model system for studying basic physical properties of the more complex biological cells.