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CHEN-SOURIAU CALCULUS FOR ROUGH LOOPS

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Abstract. We study a diffeological Calculus for rough loop spaces.

1. Introduction

Let us consider a topological space N. Let us recall what is a diffeology (see [5,24]). It is constituted of a set of maps ϕ of any open subset U of any $\mathbb{R}^n$ into N. These maps are called plots. They have to satisfy to the following requirements:

- The constant map is a plot.
- If (U, ϕ) and $(U', \phi')U \subseteq \mathbb{R}^n, U' \subseteq \mathbb{R}^n$ are two plots such that $U \cap U' = \emptyset$, then $(U \cup U', \phi \cup \phi')$ is still a plot.
- If $j : U \to U'$ is a smooth map, and (U', ϕ') is a plot, $(U, \phi' \circ j)$ is still a plot.

Let us consider as topological space the Hölder based loop space $L_{1/2-\epsilon,x}(M)$ of $1/2 - \epsilon$ Hölder maps γ from S^1 into a compact Riemannian manifold M such that $\gamma(0) = x$. $L_{1/2-\epsilon,x}(M)$ can be endowed with the Brownian bridge measure as well as the heat kernel measure (see [1]). Over it natural functionals are stochastic integrals (see [15,16,18] for the definition of stochastic integrals for the heat kernel measure).

Inspired by the considerations of Chen-Souriau, Léandre has established a differential calculus over $L_{1/2-\epsilon,x}(M)$ which allows to take derivatives of stochastic integrals. Various stochastic cohomology theories were established. The key point is that there are equal to the de Rham cohomology of the Hölder loop space. For the Brownian bridge measure, it is the purpose of [13, 14]. For the heat kernel measure, it is the purpose of [15] in the case where we replace the loop space by a torus group. As a corollary, [14] shows that a stochastic line bundle (with fiber almost surely defined) is isomorphic to a line bundle over the Hölder loop space.