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EXPLICIT PARAMETERIZATION OF EULER'S ELASTICA

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Abstract. The consideration of some non-standard parametric Lagrangian leads to a fictitious dynamical system which turns out to be equivalent to the Euler problem for finding out all possible shapes of the lamina. Integrating the respective differential equations one arrives at novel explicit parameterizations of the Euler's elastica curves. The geometry of the inflexional elastica and especially that of the figure "eight" shape is studied in some detail and the close relationship between the elastica problem and mathematical pendulum is outlined.

1. Introduction

The elastic behaviour of roads and beams which attracts a continuous attention since the time of Galileo, Bernoulli and Euler has generated recently a renewed interest in plane [2, 3, 15], space [18] and space forms [1, 11]. The first elastic problem was posed by Galileo around 1638 who asked the question about the force required to break a beam set into a wall. James (or Jakob) Bernoulli raised in 1687 the question concerning the shape of the beam and had also succeeded in solving the case of the so called rectangular elastica (second case from the top in Fig. 1). Later on in 1742 Daniel Bernoulli wrote a letter to Euler in which he had suggested to him to solve the general problem of the elastica. Following closely this suggestion Euler cast the problem in the variational form and presented the solution in an Appendix to his book on variational calculus which appeared in 1744. Euler begins his investigation with establishing the equation of static equilibrium of the "lamina" by means of the variational techniques developed in his treatise and then rederives it from mechanical principles developed earlier by James Bernoulli. The