THE SECTIONAL CURVATURE OF THE TANGENT BUNDLES WITH GENERAL NATURAL LIFTED METRICS

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Abstract. We study some properties of the tangent bundles with metrics of general natural lifted type. We consider a Riemannian manifold (M, g) and we find the conditions under which the Riemannian manifold (TM, G), where TM is the tangent bundle of M and G is the general natural lifted metric of g, has constant sectional curvature.

1. Introduction

In the geometry of the tangent bundle TM of a smooth n-dimensional Riemannian manifold (M, g) one uses several Riemannian and pseudo-Riemannian metrics, induced by the Riemannian metric g on M. Among them, we may quote the Sasaki metric, the Cheeger-Gromoll metric and the complete lift of the metric g. The possibility to consider vertical, complete and horizontal lifts on the tangent bundle TM (see [18]) leads to some interesting geometric structures, studied in the last years (see [1–3, 8, 9, 17]), and to interesting relations with some problems in Lagrangian and Hamiltonian mechanics. On the other hand, the natural lifts of g to TM (introduced in [5, 6]) induce some new Riemannian and pseudo-Riemannian geometric structures with many nice geometric properties (see [4, 5]).

Oproiu [11–13] has studied some properties of a natural lift G, of diagonal type, of the Riemannian metric g and a natural almost complex structure J of diagonal type on TM (see also [15, 16]). In [10], the same author has presented a general expression of the natural almost complex structures on TM. In the definition of the natural almost complex structure J of general type there are involved eight parameters (smooth functions of the density energy on TM). However, from the condition for J to define an almost complex structure, four of the above parameters can be expressed as (rational) functions of the other four parameters. A Riemannian metric G which is a natural lift of general type of the metric g depends on other