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ON MULTICOMPONENT MKDV EQUATIONS ON SYMMETRIC SPACES OF DIII-TYPE AND THEIR REDUCTIONS

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Abstract. New reductions for the multicomponent modified Korteveg de Vries (MMKdV) equations on the symmetric spaces of **DIII**-type are derived using the approach based on the reduction group introduced by A. Mikhailov. The relevant inverse scattering problem is studied and reduced to a Riemann-Hilbert problem. The minimal sets of scattering data $\mathcal{T}_i$, $i = 1, 2$ which allow one to reconstruct uniquely both the scattering matrix and the potential of the Lax operator are defined. The effect of the new reductions on the hierarchy of Hamiltonian structures of MMKdV and on $\mathcal{T}_i$ are studied. We illustrate our results by the MMKdV equations related to the algebra $\mathfrak{g} \simeq \mathfrak{so}(8)$ and derive several new MMKdV-type equations using group of reductions isomorphic to $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$.

1. Introduction

The **modified Korteweg de-Vries equation** (MKdV) [27]

$$q_t + q_{xxx} + 6\epsilon q_x q^2(x,t) = 0, \qquad \epsilon = \pm 1 \tag{1}$$

has natural multicomponent generalizations (MMKdV) related to the symmetric spaces [3]. They can be integrated by the ISM using the fact that they allow the following Lax representation

$$L\psi \equiv \left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x} + Q(x,t) - \lambda J \right) \psi(x,t,\lambda) = 0 \tag{2}$$

$$Q(x,t) = \begin{pmatrix} 0 & q \\ p & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

$$\begin{aligned} M\psi &\equiv \left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t} + V_0(x,t) + \lambda V_1(x,t) + \lambda^2 V_2(x,t) - 4\lambda^3 J \right) \psi(x,t,\lambda) \\ &= \psi(x,t,\lambda) C(\lambda) \end{aligned} \tag{4}$$