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AN ALGEBRAIC APPROACH TO SAXON-HUTNER THEOREM

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Abstract. Here we give some necessary and sufficient conditions for the validity of the Saxon-Hutner conjecture concerning the preservation of the energy gaps into an infinite one-dimensional lattice.

Let us consider the Schrödinger equation

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + (E - U(x))\Psi = 0 \tag{1}$$

where Ψ is the wave function, the spectral parameter E is the particle energy and $U(x)$ is a known function – the potential. Quantum mechanics deals with the above equation and its generalizations. When $U(x) = 0$ we have a free particle and when $E = k^2$, two solutions are $\mathrm{e}^{\mathrm{i}kx}$ and $\mathrm{e}^{-\mathrm{i}kx}$ representing respectively a particle moving to the right ($k > 0$) and a particle moving to the left ($k < 0$).

We will use the standard group theory notation for the invertible matrices listed below. The Lie group of pseudo-unitary matrices of signature $(1,1)$ (i.e., those 2×2 matrices having one positive and one negative square in their canonical form $\langle z, z\rangle = |z_1|^2 - |z_2|^2$), or what is the same – the group of all linear transformations of the complex plane preserving the above hermitian form $\langle\,,\rangle$ will be denoted as U(1,1) while $\mathrm{SL}(2,\mathbb{C})$ will denote the corresponding unimodular group keeping the symplectic structure $[\;,\;]$ invariant (here $[\zeta,\eta]$ is the oriented area of the parallelogram spanned on the vectors ζ, η and $\mathrm{GL}(2,\mathbb{R})$ will denote the group of all real linear transformations. We have $\langle a, b\rangle = \frac{\mathrm{i}}{2}[a,\bar{b}]$.

Proposition 1. *The intersection of any two groups coincides with the intersection of the three of them – it is the special* $(1,1)$ *unitary group* $\mathrm{SU}(1,1)$.

A monodromy operator for (1) with a finite potential is a linear operator acting on the space of states of the free particle in a special way.