HAMILTONIAN MECHANICAL SYSTEMS AND GEOMETRIC PREQUANTIZATION

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ABSTRACT

Geometric prequantization of Hamiltonian mechanical systems is discussed and some of its properties are pointed out.

1. Introduction

During the last twenty years the use of differential geometric methods in physics has undergone a steady increase. Perhaps the most extensive applications have been to the symplectic and Poisson formulation of classical mechanics and quantization. Today symplectic and Poisson geometry represents a privileged area in which both pure and applied mathematicians can enjoy fruitful cooperation.

The goal of my lecture is to present some old and new aspects in the theory of quantization of Hamilton and Hamilton-Poisson mechanical systems from the geometric quantization point of view.

2. Hamilton and Hamilton-Poisson mechanical systems

For beginning let us remind some general facts from the theory of Hamiltonian and Hamiltonian-Poisson mechanical systems which will be crucial in all that follows.

Definition 2.1

A Hamiltonian mechanical system is a triple (M, ω, H) where (M, ω) is a symplectic manifold called the phase space of the system and H is a smooth real valued function defined on M and called the energy or the Hamiltonian of the system. The evolution in time of our system is given by the integral curves of the Hamiltonian vector field X_H defined by

$$X_H \rfloor \omega + dH = 0$$

or equivalent by Hamilton's equations

$$\dot{q}^i = \frac{\partial H}{\partial p_i}$$
$$\dot{p}_i = -\frac{\partial H}{\partial q^i}$$