ON POLES OF THE ANALYTIC CONTINUATION OF INTEGRAL CURVES OF A FAMILY OF ORDINARY DIFFERENTIAL EQUATIONS

Jan M. Myszewski
Chair of Applications of Mathematics
Warsaw Academy of Agriculture
ul. Nowoursynowska 166
02-766 Warsaw, Poland

Abstract

We are concerned with the problem of investigating the properties of poles of functions which are the analytic continuation of solutions of some parametrized family of first order ordinary differential equations. We investigate the dependence of the position of poles on some parameter.

We consider a family of first order ordinary differential equations of the form:

$$z' = z^2 + bz + c \tag{1}$$

with $b, c \in \mathbf{R}$. In such a simple situation, we would like to find explicit formulas describing the relation between the numbers b, c and the position of the poles of the solutions of (1) satisfying some initial condition $z(0) = z_0$, with z_0 real. Since most of the calculations are elementary, we omit all proofs, leaving them to the reader as an easy exercise.

Observation 1

Let z_1, z_2 be solutions (not necessarily real) of the second degree polynomial equation $z^2 + bz + c = 0$. Define $d = b^2 - 4c$. Then any solution of equation (1) can be written in the following form:

(a) if $d \neq 0$ and $E(t) = \dfrac{z_0 - z_1}{z_0 - z_2} e^{t\sqrt{d}}$, then

$$z(t) = \frac{z_1 - z_2 E(t)}{1 - E(t)} \tag{2}$$

(b) if $d = 0$, then

$$z(t) = \left(\frac{b}{2} + \frac{1}{t - \frac{1}{z_0 + \frac{b}{2}}} \right) \tag{3}$$