GEOMETRIC QUANTIZATION OF STRING THEORY USING TWISTOR APPROACH

A.D. Popov and A.G. Sergeev

Steklov Mathematical Institute, Moscow
Russia

Abstract

The geometric quantization scheme for the string theory is formulated in terms of a symplectic twistor bundle over the phase manifold.

We formulate here the geometric quantization scheme for bosonic string theory based on the twistor approach. The role of phase manifold is played by the loop space ΩG of a Lie group G (for standard bosonic string, G is the group of translations of the d-dimensional Minkowski space $\mathbf{R}^{d-1,1}$). It has the structure of an infinite-dimensional pseudoKähler manifold and may be considered as an infinite-dimensional analogue of flag manifolds. We introduce a symplectic twistor bundle $\mathcal{Z} \to \Omega G$ of complex structures on ΩG compatible with symplectic structure of ΩG having the fibre $\mathcal{S} = \mathrm{Diff}(S^1)/S^1$. The latter manifold $\mathcal{S}$ is an infinite-dimensional Kähler manifold which may be considered as an infinite-dimensional analogue of the Siegel disc. This manifold (and the idea of using the twistor approach in connection with the string theory) was invented in a paper by Bowick-Rajeev [1] which was a motivation for subsequent publications on this topic.[2] We associate with the twistor space $\mathcal{Z}$ a double fibration

$$\begin{array}{ccc} & \mathcal{Z} & \\ {}^{\pi}\swarrow & & \searrow_{p} \\ \Omega G & & \mathcal{S} \end{array}$$

where $p : \mathcal{Z} \to \mathcal{S}$ is a natural holomorphic projection. Similar double fibrations arise also in the conventional twistor theory.[3]

The standard geometric quantization scheme is interpreted in terms of this double fibration roughly as follows. We pull back the prequantum bundle L over ΩG to a holomorphic line bundle $\tilde{L} \to \mathcal{Z}$. Projective representations of the Lie algebra $\mathrm{Vect}(S^1)$ of the group $\mathrm{Diff}(S^1)$ of diffeomorphisms of the circle in polarized Fock spaces generate a connection on a Fock bundle $\tilde{H} \to \mathcal{S}$ with the fibre $\tilde{H}_J$ in a point $J \in \mathcal{S}$ given by holomorphic sections of $\tilde{L}$ over $p^{-1}(J) \subset \mathcal{Z}$. To quantize ΩG in twistor terms means to construct a quantization bundle $\tilde{\mathcal{H}} \to \mathcal{S}$ with a flat unitary connection on it. We cannot use for $\tilde{\mathcal{H}}$ the Fock bundle $\tilde{H}$ with the connection generated by projective representations of $\mathrm{Vect}(S^1)$ because it is never flat. The right quantization bundle is